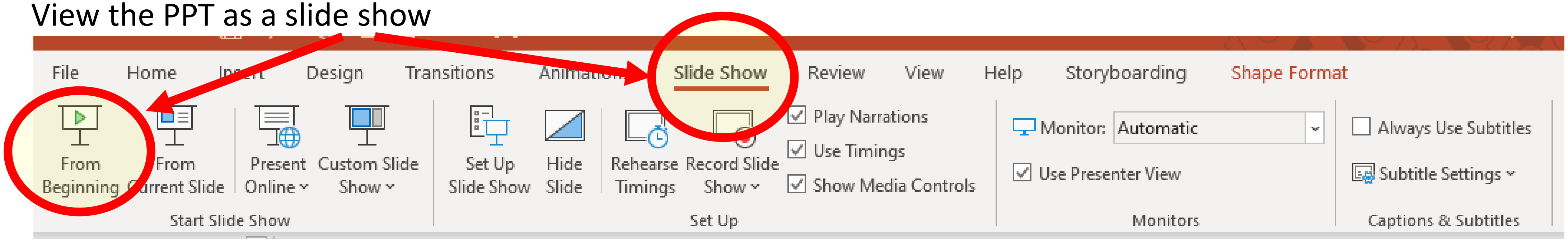
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.4a

Adding and Subtracing Rational Expressions

Today you will:

- Add and Subtract rational expressions
- Oh yeah … add and subtract fractions! Doesn’t get any better than this! ;)
- Practice using English to describe math processes and equations

Core Vocabulary:

- Rational expression, p. 376
- LCD (Lowest Common Denominator)

Prior:

- Fractions and fraction arithmetic
- Polynomials
- LCM (Lowest Common Multiple) … yeah LCD is just LCM for fractions …

Today we are going to add and subtract *Rational Expressions*

The main things to remember…

1. You can think of subtraction as adding the negative.
2. You can ***ONLY*** add fractions that have a common denominator.
3. You can ***ONLY*** add fractions that have a common denominator.
4. You can ***ONLY*** add fractions that have a common denominator.
5. …and did I mention…
6. You can ***ONLY*** add fractions that have a common denominator.

You can ***ONLY*** add fractions that have a common denominator

Adding *Rational Expressions* that have the same denominator

I think we can all agree that if two fractions have the ***same*** denominator, they have ***common denominators***

Examples:

- $\frac{1}{5} + \frac{3}{5}$
- $\frac{x}{2} + \frac{3}{2}$
- $\frac{7}{4x} + \frac{3}{4x}$
- $\frac{2x}{x+6} - \frac{5}{x+6}$

So all the above have ***common (same) denominators*** … in each case the two denominators are identical

Adding *Rational Expressions* that have the same denominator

Examples:

- $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5}$
- $\frac{x}{2} + \frac{3}{2} = \frac{x+3}{2}$
- $\frac{7}{4x} + \frac{3}{4x} = \frac{7+3}{4x}$
- $\frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$

If the fractions have the same/identical denominator:

1. add (subtract) the numerators
2. keep the denominator
3. …and finally simplify

Note: in these examples there is nothing to simplify…

a. $\frac{7}{4x} + \frac{3}{4x} = \frac{7 + 3}{4x} = \frac{10}{4x} = \frac{5}{2x}$ Add numerators and simplify.

b. $\frac{2x}{x + 6} - \frac{5}{x + 6} = \frac{2x - 5}{x + 6}$ Subtract numerators.

Adding *Rational Expressions* that have *DIFFERENT* denominators

Easy-peasy to add if they have the same denominators … what do we do if they have different denominators?

Well, the trick is … you ready for this? … morph the problem so they have the same denominators.

- Find the LCD … Least Common Denominator
- LCD = LCM (Least Common Multiple) for the two denominators

So we need to remember how to find the LCM…

To get there, let's try this…

$\times 1$

$$\frac{6}{7x}\cdot\left(\frac{2x}{2x}\right)=\frac{12x}{14x^2}$$

Ask yourself: what is missing?

For the numerator: what do we need to multiply 6 by to get $12x$?

- Easy: $6 \cdot 2x = 12x$

For the denominator: what do we need to multiply $7x$ by to get $14x^2$?

- Easy: $7x \cdot 2x = 14x^2$

Note that we multiplied by $\frac{2x}{2x}$ which is 1 … so the 1st fraction is the same as the 2nd

… which is kind of a "duh" cause that's what = means right?

To get there, let's try this...

Ask yourself: what is missing?

$\times 1$

$$\frac{3a}{5b} \cdot \left(\frac{7bc}{7bc}\right) = \frac{21abc}{35b^2c}$$

For the numerator: what do we need to multiply 3a by to get $21abc$?

- Easy: $3a \cdot 7bc = 21abc$

For the denominator: what do we need to multiply $5b$ by to get $35b^2c$?

- Easy: $5b \cdot 7bc = 35b^2c$

Note that we multiplied by $\frac{7bc}{7bc}$ which is 1 ... so again the 1st fraction is the same as the 2nd

To get there, let’s try this…

Ask yourself: what is missing?

$(x+3)\cdot(x \; ? \; 7) = (x^2 - 4x - 21)$

$(x+3)\cdot(x - 7) = (x+3)(x-7)$

To compare them we need to factor the quadratic on the right side

- …to keep track, it was $x^2 - 4x - 21$
- Now we can see both sides have the factor $(x+3)$ in common
- And the left side is missing the factor $(x-7)$

Here we needed to factor to find what is missing.

This is what you have to do to find the Lowest Common Multiple:

1. Factor each side so you can see all the necessary parts
2. Figure out what is missing

Okay, we have the “what is missing?” skill down, how does that help us with finding the LCM?

Definition:

- The **LCM (lowest common multiple)** of two numbers is the ***smallest*** number they both divide evenly into.

Simple example: what is the LCM of 4 and 10? In math we write this as LCM(4, 10).

- One method is list the multiples of each and find the first match:
  - Multiples of 4: 4, 8, 12, 16, 20 … (stopping there because we know 20 is a multiple of 10!)
  - Multiples of 10: 10, 20
  - The LCM of 4 and 10 is 20.
- You may be tempted to just multiply the two numbers together to find the LCM.
  - Problem is while that will be a multiple, it likely won’t be the lowest multiple.
  - For this problem, 4 x 10 = 40 ... which isn’t the lowest multiple.

Okay, we have the "what is missing?" skill down, how does that help us with finding the LCM?

Another example: what is the LCM(15, 42)?

- Okay, I'm going to stop us right there…
- I for one, don't want to list out a bazillion multiples … the answer for this one is 210
- Is there another way? Let me answer the question with a few questions:
  - What are the prime factors of 15? 15 = 3 x 5
  - What are the prime factors of 42? 42 = 6 x 7 = 2 x 3 x 7
  - What are the prime factors of 210? 210 = 2 x 105 = 2 x 5 x 21 = 2 x 5 x 3 x 7 = 2 x 3 x 5 x 7
  - What do you notice? Lots of similarity!
    - The prime factors of 210 are a combo of the prime factors of 15 and 42
  - Look at the prime factors for 15 and 42 … what are they each missing that the other has?
    - 15 is missing the 2 and 7
    - 42 is missing the 5

for 15: 3 x 5 x 2 x 7 = 2 x 3 x 5 x 7 = 210

for 42: 2 x 3 x 7 x 5 = 2 x 3 x 5 x 7 = 210

Add the missing pieces and you have the LCM!

Let's make sure we got this …

What is the LCM (lowest common multiple) of $x^2 - x - 12$ and $12x - 48$?

1. Factor each expression completely (break each down to its prime factors):
   - $x^2 - x - 12 = (x + 3)(x - 4)$
   - $12x - 48 = 12(x - 4)$

Note: we could further break 12 down but the other expression does not have a constant factor so no need.

2. Determine what is missing in each:
   - $(x + 3)(x - 4)$ is missing the 12 so combined we have $12(x + 3)(x - 4)$
   - $12(x - 4)$ is missing the $(x + 3)$ so combined we have $12(x + 3)(x - 4)$
3. Make sure both are the same … the result is the LCM:
   - $12(x + 3)(x - 4)$

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

SOLUTION

Step 1 Factor each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

Step 2 The LCM is the product of the highest power of each factor that appears in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

This means you have to account for every factor and make sure the result has one of everything.

Review/Recap

- To add or subtract rational expressions you ***MUST*** have common denominators.
  - Add (or subtract) the numerators
  - Keep the denominator
  - Always simplify
- If the rational expressions have different denominators:
  - You need to make them the same … but without changing the fractions
  - Do this by finding the LCD (lowest common denominator)
  - …which is another way of saying “find the LCM (lowest common multiple) of the denominators
- To find the LCM of two expressions:
  1. Factor each expression completely
  2. Determine what is missing from each
  3. The LCM will have one of each factor:
     1. Combine the missing parts for each
     2. Make sure both are the same

Homework

Pg 388, #3-16